647510 **BSCalgCC5010** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination

Oct/Nov. - 2022(Old Course)

MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any one of the following questions. **[07]**

1. Show that closure of a set is closed in a metric space.
2. Prove that arbitrary union of open sets is open in a metric space. Does the result remain true for closed sets? Justify.

Q.1 (B) Answer any one of the following questions. **[04]**

1. Prove that every subset of a discrete metric space is both open and closed.
2. Define dense set and in usual notations prove that Q is dense R.

Q.1 (C) Answer any one of the following questions. **[03]**

1. State and prove Hausdorff principle.
2. Let (X, d) be a metric space and $A, B \subset X$. Is it true that $A' \cap B' = (A \cap B)'$? Justify.

Q. 2 (A) Answer any one of the following questions. **[07]**

1. Show that the function $f(x) = 2x^2 + 1$ is R-integrable on [0, 3] and evaluate $\int_0^3 2x^2 + 1\, dx$.
2. Let f and g be bounded R-integrable functions on *[a, b]*. Show that $f \cdot g$ is also R-integrable on *[a, b]*.

Q.2 (B) Answer any one of the following questions. **[04]**

1. Let $P = \left\{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ be a partition of $[0, \frac{\pi}{2}]$ and $f(x) = \cos x$. Find $U(P, f)$ and $L\,(P, f)$.
2. Let f be a bounded function on $[a, b]$ and P and P^* be two partitions of $[a, b]$ such that $P \subset P^*$. Then show that $U(P^*, f) \leq U(P, f)$.

Q.2 (C) Answer any one of the following questions. **[03]**

1. Write an example to show that a bounded function may not be R-integrable.
2. Define upper Reimann integral and lower Reimann integral.

Q.3 (A) Answer any one of the following questions. **[07]**

1. Verify general form of first mean value theorem for $f(x) = x$ and $g(x) = e^x$ on [-1, 1].
2. Let f be a bounded R-integrable function on *[a, b]* and F be a function defined as $F(x) = \int_a^x f(t)\ dt, a \leq x \leq b$. Then prove that F is continuous on *[a, b]*. Further, if f is continuous on *[a, b]*, then F is differentiable on *(a, b)* and $F'(x) = f(x), \forall x \in (a, b)$.

Q.3 (B) Answer any one of the following questions. **[04]**

1. State and prove fundamental theorem of integration.
2. Let S be a set containing n elements. In usual notations prove that S_n is a group with respect to the binary operation of composition of mappings.

Q.3 (C) Answer any one of the following questions. **[03]**

1. Evaluate : $\lim\limits_{n \to \infty} \frac{1}{n^2} \sum_{r=0}^{n-1} \sqrt{n^2 - r^2}$.
2. Determine the number of binary operations that can be defined on the set $S = A \times B$, Where A, B are two sets with $|A| = 3, \& \ |B| = 2$.

Q.4 (A) Answer any one of the following questions. **[07]**

1. Prove that intersection of two subgroups of a group G is a subgroup of G but union of two subgroups may not be a subgroup of G.
2. Prove that order of a permutation is equal to the LCM of the length of its disjoint cycles.

Q.4 (B) Answer any one of the following questions. **[04]**

1. Prove that every permutation $f \in S_n$ can be expressed as a composition of disjoint cycles.
2. State and prove Lagrange's theorem.

Q.4 (C) Answer any one of the following questions. **[03]**

1. Using Fermat's theorem, find the remainder when, 8^{2014} is divided by 13.
2. Prove that a non-cyclic group has a proper subgroup always.

Q.5 (A) Answer any one of the following questions. **[07]**

1. State and prove necessary and sufficient condition for a subgroup of a group to be normal.
2. State and prove Cayley's theorem.

Q.5 (B) Answer any one of the following questions. **[04]**

1. Show that isomorphism of groups is a symmetric relation.
2. Define center of a group G and show that it is a subgroup of G.

Q.5 (C) Answer any one of the following questions. **[03]**

1. Let $(G, *)$ and (G', Δ) be groups and $\phi: G \to G'$ be an isomorphism. If G is cyclic then show that G' is also cyclic.
2. If H and K are normal subgroups of a group G such that $K \cap H = \{ e \}$, then show that $hk = kh \,; \forall\, h \in H, k \in K$.

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